DEFINITION: A real number x is a perfect square if and only if $x = y^2$ for some integer y.

OR MORE FORMALLY

For all real numbers x, x is a perfect square if and only if there exists an integer y such that $x = y^2$

"THEOREM": For all real numbers r, if r is a perfect square, then $4r$ is a perfect square.

OR MORE CASUALLY

If r is a perfect square, then $4r$ is a perfect square.

PROOF: Let r be a particular but arbitrarily chosen real number such that r is a perfect square.

By the definition of "perfect square", $r = t^2$ for some integer t.

So, $4r = 4t^2 = (2t)^2$, and $2t$ is an integer since the product of integers is an integer.

Therefore, $4r$ is a perfect square.

This is the style of many proofs.
However, implied by this is a sequence of universal instantiation and universal modus ponens arguments which are not explicitly stated.

"THEOREM": For all real numbers r, if r is a perfect square, then $4r$ is a perfect square.

PROOF: Let r be a particular but arbitrarily chosen real number such that r is a perfect square.

Using definition of perfect square

For all real numbers x, if x is a perfect square, then there exists an integer y such that $x = y^2$.

r is a perfect square (and r is a real number).

Therefore, there exists an integer t such that $r = t^2$. **(UNIVERSAL MODUS PONENS)**

Using multiplication property of equality

For all real numbers a, b and c, if $b = c$, then $ab = ac$.

$r = t^2$ (and 4, r and t^2 are real numbers).

Therefore, $4r = 4t^2$. **(UNIVERSAL MODUS PONENS)**

Using distributive rule of exponentiation over multiplication

For all real numbers a and b, and all positive integers n, $a^n b^n = (ab)^n$.

2 and t are real numbers, and 2 is a positive integer.

Therefore, $4t^2 = (2t)^2$. **(UNIVERSAL INSTANTIATION)**

Using transitive law of equality

For all real numbers a, b and c, if $a = b$ and $b = c$, then $a = c$.

$4r = 4t^2$ and $4t^2 = (2t)^2$ (and $4r$, $4t^2$ and $(2t)^2$ are real numbers).

Therefore, $4r = (2t)^2$. **(UNIVERSAL MODUS PONENS)**

Using closure property of integers under multiplication

For all integers a and b, ab is an integer.

2 and t are integers.

Therefore, $2t$ is an integer. **(UNIVERSAL INSTANTIATION)**

Using definition of perfect square

For all real numbers x, if there exists an integer y such that $x = y^2$, then x is a perfect square.

$4r = (2t)^2$ and $2t$ is an integer (and $4r$ is a real number).

Therefore, $4r$ is a perfect square. **(UNIVERSAL MODUS PONENS)**